公開セミナー

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場所：理学部 A510 セミナー室
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題目：**Natural extensions for digital expansions**
(デジタル表現の自然拡大)

We consider two classes of transformations providing digital expansions of real numbers.

The first class consists of piecewise linear transformations T with constant slope $\beta > 1$, such as $x \mapsto \beta x \mod 1$, which provides the greedy β-expansions. These transformations are clearly not invertible, but if β is a Pisot number of degree d and the digits are in $\mathbb{Q}(\beta)$, then there exists a canonical way to define an invertible transformation on $\mathbb{R}^d$ which admits T as a factor. This invertible transformation is called a natural extension of T. The natural extension domain has fractal boundary if $d > 2$. Closely associated to it is a multiple tiling of $\mathbb{R}^{d-1}$, as defined by Akiyama. A version of the Pisot conjecture states that this multiple tiling is always a tiling if we consider transformations $x \mapsto \beta x \mod 1$ with Pisot units β. Surprisingly, this property is not true for slightly more general transformations. E.g., the transformation $x \mapsto \beta x - \lfloor \beta x + 1/2 \rfloor$ gives a tiling if β is the golden mean, but a double tiling if β is the Tribonacci number or the smallest Pisot number.

The second class consists of Nakada's α-continued fraction transformations. Here, the natural extension domain is composed of a finite number of rectangles provided that α is in $[\sqrt{2} - 1, 1]$. For $\alpha < \sqrt{2} - 1$, the domain has again a fractal structure. The case $\alpha = 1/r$ was described recently by Luzzi and Marmi. Nakada and Natsui obtained interesting results on the entropy of the transformations. We discuss the natural extensions for general α and exhibit similar phenomena as in the case of generalized β-transformations.

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